

# Modeling Basic Aspects of Cyber-Physical Systems

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**Abstract**—Designing novel cyber-physical systems entails significant, costly physical experimentation. Simulation tools can enable the virtualization of experiments. Unfortunately, current tools have shortcomings that limit their utility for virtual experimentation. Language research can be especially helpful in addressing many of these problems. As a first step in this direction, we consider the question of determining what language features are needed to model cyber-physical systems. Using a series of elementary examples of cyber-physical systems, we reflect on the extent to which a small, experimental domain-specific formalism called Acumen suffices for this purpose.

## I. INTRODUCTION

Increasing the computational power of everyday products is revolutionizing the way we live. Segways can move us from one location to another without any need for large, cumbersome, or noisy vehicles. Cars can park themselves, and warn us when we are changing lanes unsafely. The future appears more fantastic than science fictions depicted it just a few years ago. At the same time, the increasingly tight coupling between computational and physical mechanisms, often described as cyber-physical systems (CPS), is creating a challenge for the traditional product development cycles. For example, car manufacturers are concerned about the amount of physical testing necessary to assure the safety of a car with a high degree of autonomy.

Since the beginning of time, physical testing has been the basis for justified true belief in the qualities of a new product. A key ingredient of physical testing is having a collection of specific usage scenarios. But the presence of even simple computational components can make it make it difficult to identify enough usage scenarios to exercise more than a minute fraction of the possible behaviors of the system. These observations are spurring the developers of cyber-physical systems to rethink the traditional methods and processes for developing and testing new products.

### A. Virtual Experiments and Language Research

One way to alleviate the testing problem is to use computer simulations [1] to perform virtual experiments [2]. Virtual testing can be used to quickly eliminate obviously bad designs. It can also help build confidence that a new design can pass test scenarios developed by an independent party [3]. Creating a framework for conducting virtual experiments requires a concerted, interdisciplinary community effort to address a wide range of challenges, including:

- 1) Educating designers in the cyber-physical aspects of the products they will develop, both in terms of:
  - How they these aspects are modeled, and
  - What types of system-level behaviors they generate.
- 2) Developing expressive, efficient, and robust modeling and simulation tools to support the innovation process. It is particularly important that the underlying models are:
  - Easy to understand and analyze at each stage of the design process, and
  - Easy to reason about across stages.
- 3) Accumulating extensive libraries of component models that are both
  - Grounded in physical principles and analytic methods, and
  - Validated experimentally.

All three challenges would benefit from better language-based technologies for describing and simulating cyber-physical systems. Engineering methods centered around a notion of executable or *effectively-computable* models can have profound positive impact on the pace of advancement of knowledge and engineering practice in cyber-physical systems.

1) *The Educational Challenge*: For decades, engineering and science education has focused on providing specialized training within well-defined disciplines. As a result, to design an advanced cyber-physical system such as a robot, we must engage several experts with advanced degrees from a number of different disciplines, such as mechanical engineering, electrical engineering, computer science, and biology. Not only does this recurring task make it difficult to assemble a team with necessary expertise for a project, the team may still lack a common language for discussing key issues that are fundamental to the design of robotic systems, but that are treated differently across disciplines.

Addressing the educational challenge will require a concerted effort to break down artificial boundaries between disciplines. A key step towards achieving this goal will be to find a *lingua franca* (or “common language”) to communicate about fundamental issues that recur in the development of a variety of different cyber-physical systems. Part of such a language will be a jargon for communicating efficiently among experts; part will be an appropriate mathematical formalism. Language research can be particularly helpful in developing tools that are closely aligned with executable subsets (c.f. [4], [5]) of mathematical notations that are

already used by many engineers and scientists but are not available in mainstream programming languages and tools.

2) *The Modeling and Simulation Challenge*: The modeling and simulation challenge can also be approached from a linguistic point of view. Precise reasoning about models during each stage of the process can be improved by applying classical (programming) language design principles, including defining a formal semantics. Reasoning across stages can be facilitated by using two ideas from language design: 1) increasing the expressivity of a language to support multiple stages in the design process, and 2) automatically compiling models from one stage to the next to reduce manual work and opportunities for mistakes as models are translated from one stage of the design process to the next.

3) *The Modeling Library Challenge*: It seems reasonable to expect that the dynamics surrounding the development and use of modeling libraries can be similar to those for software libraries. We can envision CPS design as a community process where there are library providers and library consumers, and where different libraries and test suites are used to benchmark and evaluate various offerings. In such a setting, the interfaces between components become important, motivating questions related to advanced linguistic techniques such as types, static checking, unit testing, contracts, assertions, assume-guarantee reasoning, and blame assignment, all of which can be expected to play a key role in addressing this challenge.

### B. A Small, Experimental Language for Hybrid Modeling

To better understand the core linguistic issues that arise in addressing these challenges, we are developing a modeling language called Acumen [6], [7]. A key characteristic of modeling and simulation languages for cyber-physical systems is supporting hybrid (continuous/discrete) mathematical models [1]. Modelica [8] and SimScape are widely used examples of such languages. Hybrid modeling can be supported using a small number of constructs, namely:

- Ground values (e.g., True, 5, 1.3, "Hello")
- Vectors and matrices (e.g., [1, 2], [[1, 2], [3, 4]])
- Object definition (`class C (x, y, z) ... end`)
- Object instantiation and termination (`create, terminate`)
- Variable declarations (including a special variable called `_3D` for generating visualizations) (`private ... end`)
- Variable derivatives (`x', x'', ...`)
- Continuous assignment (`[=]`)
- Discrete assignment (`=`)
- Conditional statements (`if`, and `switch`)
- Expressions and operators on reals (`+`, `-`, `...`)

It appears that a language with just these features can be helpful in addressing the educational challenge. For example, we used such a language for a term-long project in an eight week course on cyber-physical systems [9], and seems to have been received positively. It appears that being able to concretely explain a wide range of concerns in a small

language has two key benefits. The first relates to CPS education. Using a small language can help highlight the connections between different concepts, and avoid the introduction of artificial distinctions between manifestations of the same concept in different contexts. The second relates to language and tools research. Showing how such a wide range of cyber-physical phenomena can be captured in a small language helps emphasize the expressivity of such a small language, and provide a basis for arguing against the introduction of additional language features until a compelling case for the addition of such language features has been made.

These observations inspired us to step back from the details of that particular teaching experience to reflect on two questions relating to the challenges described above. First, what can we view as the basic cyber-physical aspects of a robot system? Second, how well can they be explained in terms of a small hybrid-systems modeling formalism?

### C. Contributions

This paper considers several aspects of cyber-physical systems that can be seen as common features of cyber-physical systems, and uses them to reflect on how they can be expressed and illustrated using the small hybrid systems modeling language introduced above.

Visual and geometric presentation is a critical aspect of analytical modeling that can hide in plain sight (Section II). Technically, it is not part of analytical modeling, but it is indispensable for efficiently understanding both the specification and the results of a virtual experiment. From a pedagogic point of view, the trigonometric reasoning involved in creating visualizations provides a natural path into geometry of motion (kinematics). Basic mechanics and dynamics come next (Section III), and a range of analytical principles used to model physical systems. They also motivate the use of differential equations, which in turn provide much of the background needed to motivate the discussion of control (Section V). To help experimentally evaluate the effect of control, it is useful to consider mechanisms for modeling disturbances (Section VI). By introducing these aspects, we are able to present the simplest possible example of how to model and test a cyber-physical system. This allows us to return back to the physical component and refined it. A natural way to do this in the robotics domain is to use ideas from rigid body dynamics (Section VII). Similarly, we can refine the model of the control system by capturing the way in which implementation on a digital computer introduces both discretization and quantization effects (Section VIII).

After the discussion of the individual aspects has been considered, we summarize our observations about the language (Section IX) and conclude.

## II. VISUAL AND GEOMETRIC PRESENTATION

Visual presentation plays an essential role in the design of cyber-physical systems. For many people, it is hard to imagine a robot (and possibly any other design, for that matter) without conjuring an image of a general physical form. If we want to replace physical prototyping with virtual

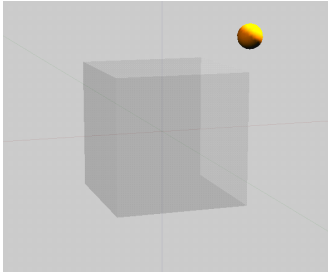


Fig. 1. The 3D output generated for an instance of the class sphere.

prototyping, visualization becomes a necessity. From the educational point of view, this can be serendipitous, because it can provide an opportunity to introduce trigonometry, which itself is needed to model geometric features of objects as well as to work with both the kinematics and dynamics of physical objects.

#### A. Drawing 3D Objects

A small language for hybrid modeling and simulation can be easily extended with a lightweight mechanism for three dimensionals (3D) visualization [10]. In Acumen, the user can specify 3D visualizations through a special variable called `_3D`. This variable is only special in that it is read by the implementation and used to generate a dynamic 3D scene. In principle, any graphical rendering technology can be used by an implementation to realize these visualizations. In practice, the current implementation used the Java3D library, which is built on top of OpenGL.

#### B. Class Definitions and Parameterization

The following class definition specifies a particular way for drawing a sphere:

```
class sphere (m,D)
  private
    p =[0,0,1];
    _3D = [{"Sphere", D+[0,0,1],
           0.03*sqrt(m),
           [m/3,2+sin(m),2-m/2],
           [1,1,1]]];
  end
  _3D [=] [{"Sphere", D+p,
           0.03*sqrt(m),
           [m/3,2+sin(m),2-m/2],
           [1,1,1]]];
end
```

The class parameter `m` represents a mass. This parameter is only used to pick a size and a color for the sphere. The parameter `D` is a display reference point. Passing different `D` values to individual objects facilitates creating visualizations where the individual objects appear in different places. The private section declares local variables as well as their initial value at the (simulated) time when the object is created. The variable `p` is used to represent the position of the sphere.

The private section and the main body of the class definition contain similar expressions for `_3D`. Both expressions consist of a vector that has a format understood by the 3D visualization part of the Acumen implementation. The first definition is a discrete assignment that happens only at object creation time. The second expression is a continuous assignment that is computed all the time as long as the object exists in the simulation.<sup>1</sup> The format of the vector is as follows: The first field is a string indicating that the shape we want is a sphere. The second field is the coordinate for the center of the sphere. The next field is the radius. Here we compute chose to make the radius a simple function of the mass. This function is not intended to have any physical meaning. Rather, to produce reasonable effects for the examples presented in this paper. The next field contains a vector that represents the red/green/blue (RGB) colors for this sphere. To help us distinguish different objects, we have again used an ad hoc formula to generate a color based on the mass passed in. The last field can be used to express an orientation, and only matters when the sphere has a texture. Figure 1 depicts a visualization generated using this class.

#### C. Object Creation, Continuous Assignment, and Animation

We can create sphere by writing “`s = create sphere (5,[0,0,0])`” in the initialization section and then “`s.p [=] [0.1, 0.2, 0.3]`”. To generate 3D animations, all we have to do is to let the value of “`p`” vary over time, as in the following code:

```
class moving_sphere (m,D)
  private s = create sphere (m,D);
          t = 0; t' = 0
  end
  t' [=] 5;
  s.p [=] [sin(t)*sqrt(1-(sin(t/10)^2)),
          cos(t)*sqrt(1-(sin(t/10)^2)),
          sin(t/10)];
end
```

Here the variable `t` and its derivative `t'` are introduced here to model a local variable that progresses at exactly five times the rate of time. All that is needed to do that is to include the equation “`t' [=] 5`”. The time-varying variable `t` is then used to generate some interesting values for the `x`, `y`, and `z` components of the the position field `p` that represents the center of the sphere object `s`.

As noted earlier, we can have instances of the same object (such as the entire `moving_sphere` example) appear at different parts on the screen by varying the `D` parameter. By changing the value of the position parameter `p`, we can create an animation with two spheres moving in a synchronized fashion.

It is useful to note that a 3D visualization facility can also be used to visualize not only 3D values but also scalar values. For example, it is useful to define objects that assist in visualizing specific scalar values during a simulation. The

<sup>1</sup>Initialization is cumbersome in the current syntax for Acumen, as it requires using two very similar expressions.

following class defines a class to visualize a scalar value as a cylinder of length proportional to that value:

```
class display_bar (v,c,D)
  private
    _3D = ["Cylinder", D+[0,0.2,0],
          [0.02,v], c,
          [-3.14159265359/2,0,0]]
  end
  _3D = ["Cylinder", D+[0,0.2,v/2],
        [0.02,v], c,
        [-3.14159265359/2,0,0]];
end
```

Following the string “Cylinder”, the first value represents the center of the cylinder. We take this to be  $v/2$  because this will allow us to keep one end of the cylinder fixed as the value of  $v$  changes. The next parameter is a tuple containing the radius and length of the cylinder. The next parameter is color. The last parameter specifies orientation angles for the cylinder. A screenshot of an instance of this class will be presented shortly.

#### D. Vector and Trigonometric Calculation

In many cases, it is necessary to perform a bit of geometrical calculation to create the desired shape. The need for such calculations can arise in situations that may be simpler than expected. An example of such a situation is drawing a cylinder between two points. Often, this cannot be done directly because many underlying visualization tools do not describe cylinders directly in this exact manner. Rather, it is common to use two angles that specify the orientation of the cylinder. Once we have figured out all necessary calculations, they can be encapsulated in one class as follows:

```
class cylinder (D)
  private
    p =[0,0,0]; q=[0,0,0];
    _3D = [{"Cylinder", D, [0,0],
           [0,0,0],[0,0,0]]];
    radius = 0.01; length = 0.01; alpha=0;
    theta= 3.14159265359/2;
    x=0;y=0;z=0
  end
  x [=] dot(p-q,[1,0,0]);
  y [=] dot(p-q,[0,1,0]);
  z [=] dot(p-q,[0,0,1]);
  length [=] norm(p-q);
  alpha [=] asin(z/length);
  if (y>0)
    theta [=] asin(x/(length*cos(alpha)))
  else
    theta [=] -asin(x/(length*cos(alpha)))
    +3.14159265359
  end
  _3D [=] [{"Cylinder", (p+q)/2+D,
             [radius,length],
             [1,1,1],[alpha,0,-theta]]];
end
```

The operators `dot` and `norm` operators compute the dot product and the vector norm (or length). Creating such an object is a good first exercise in coordinate transformation. Versatility with such transformations is an important skill for working with physical systems both in terms of Newtonian modeling as well as other, more advanced modeling techniques (c.f. [5]). Because they are executable models that produce visual results, developing small, purely graphical objects such as the ones above can be a gratifying way for students to learn about and practice the necessary geometric and necessary steps to understand how other aspects of robot mechanics and motion are modeled.

### III. MECHANICS AND DYNAMICS

In contrast to the effort needed to describe geometric and visual objects, describing basic mechanical systems and their dynamics can be done more concisely. A point mass that can only move long dimension can be represented as follows:

```
class mass_1d (m,p0,D)
  private
    p=p0; p'=0; p''=0; f=0; e_k=0;
    s=create sphere (m,D)
  end
  p'' [=] f/m;
  e_k [=] 0.5 * m * (p')^2;
  s.p [=] [0,0,p]
end
```

The object takes as parameters a mass  $m$ , an initial position  $p_0$ , and a reference point for visualization.<sup>2</sup> Internally, the mass keeps track of a position  $p$ , its first and second derivatives  $p'$  and  $p''$ , a force  $f$ , and the kinetic energy  $e_k$ . For visualization, a sphere object is created during initialization. The body of the class definition specifies that the acceleration of the object,  $p''$ , is determined by Newton’s law  $F = ma$ , where we are solving for acceleration (which is just  $p''$  here). The expression for energy uses the built-in dot-product operation on vectors. Finally, we set the position  $p$  of the visual object sphere to be the same as the position  $p$  of the current object.

Supporting vector operations make it possible to define a similar object that has a three dimensional position almost just as simply:

```
class mass (m,p0,D)
  private
    p=p0; p'=[0,0,0]; p''=[0,0,0];
    f=[0,0,0]; e_k=0;
    s = create sphere (m,D);
  end
  p'' [=] f/m;
  e_k [=] 0.5 * m * (dot(p',p'))^2;
  s.p [=] p;
end
```

<sup>2</sup>For reasons of space, this paper uses short (often single-character) variable names. While this is closer to mainstream mathematical notation, in larger models it may be better style to use longer names for variables.

Note that it is convenient in this domain to have derivatives over vectors. We can induce continuous behaviors in such an object by mean of an external continuous assignment. For example, the effect of a gravitational force on a mass object  $m$  by a continuous assignment “ $m.f [=] m.m*[0,0,-9.81]$ ”.

An idealized, 3D spring can be modeled as follows:

```
class spring (k,l0,D)
  private p1=[0,0,0]; p2=[0,0,0];
          f1=[0,0,0]; f2=[0,0,0];
          dl = [0,0,0]; e_p=0;
end
dl  [=] p2-p1 * (1-l0/norm(p2-p1));
f1  [=] k*dl;
f2  [=] -k*dl;
e_p [=] 0.5 * k * dot (dl,dl);
end
```

This class associates a different force with each end of the spring, and that computes only a potential energy  $e_p$  rather than a kinetic energy. No visualization is included in this object, but that can be easily done using techniques presented above.

#### A. Impacts and Discrete Assignment

An important physical effect in dynamics is impact. Often, it is convenient to model impacts as a sudden effect. Discrete assignments can be used for this purpose. The following model provides an example of the use of discrete assignment to model the impact of a falling ball with a floor:

```
class bouncing_ball (D)
  private
    m = create mass_1d (10, 3, D);
    bk = create display_bar
          (0, [3,0.2,0.2],D+[ 0.1,0.2,0]);
    bp = create display_bar
          (0, [0.2,3,0.2],D+[-0.1,0.2,0]);
    bt = create display_bar
          (0, [0.2,0.2,3],D+[ 0,0.2,0]);
  end
  m.f [=] m.m * -9.81;
  if (m.p < 0 && m.p' < 0)
    m.p' = -0.9 * m.p'
  end;
  bk.v [=] m.e_k / (m.m * 9.81);
  bp.v [=] (m.m * 9.81 * m.p)
           / (m.m * 9.81);
  bt.v [=] bk.v + bp.v;
end
```

The model uses the mass class along with a continuous gravity model and a ground-impact model where the ball loses 10% of its velocity. The class `display_bar` is used to display colored bars to present some additional information in the 3D output. The mass model used here has only one degree of freedom along the Z axis. We use three display bars to visually represent the kinetic and potential energy, as well as their sum. The discrete assignment occurs inside

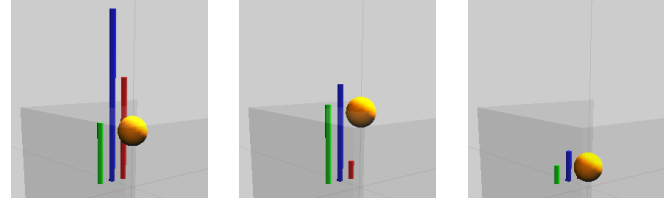
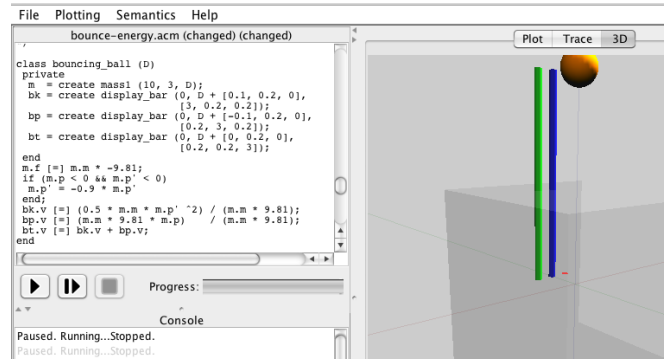


Fig. 2. The Acumen IDE with the bouncing ball model and simulation results. The green bar indicates the potential energy, the red one is the kinetic energy, and the blue bar is their sum. The total energy decreases with each ground impact, and during the free flight phase the two energies behave as expected.

the `if` statement that detects impact with the ground plane. Figure 2 shows a sequence of screenshots, one including the Integrated Development Environment (IDE), which results from running this example. It can be seen that, as expected, the total energy decreases at each impact, while the kinetic and potential energies reach their respective maxima and minima at the height of the bounce and the impact at ground level.

Now we turn to creating systems made from components such as the mass and spring components that we have just introduced.

#### IV. CAPTURING CONCEPTUAL STRUCTURE: OBJECT BOUNDARIES AND COMPOSITION

A benefit of using classes in defining a model is that it helps us think clearly about the conceptual boundary between the different components that we are modeling. Connecting components is a matter of relating fields in different components though continuous assignments. For example, the following class models a system consisting of three masses connected by two springs:

```
class example_3 (D)
  private
    m1 = create mass (15,[0,0, 1],D);
    m2 = create mass (5, [0,0,-1],D);
    m3 = create mass (1, [0,0,-1.5],D);
    s1 = create spring (5,1.75,D);
    s2 = create spring (5,0.5,D);
    b = create display_bar (-1.5,0,D)
  end
  s1.p1 [=] m1.p; s1.p2 [=] m2.p;
  s2.p1 [=] m2.p; s2.p2 [=] m3.p;
```

```

m1.f  [=] s1.f1;
m2.f  [=] s1.f2 + s2.f1;
m3.f  [=] s2.f2;
b.v   [=] (m1.e_k + m2.e_k + m3.e_k
           + s1.e_p + s2.e_p)*12;
end

```

The class uses an instance of the class `display_bar` to draw a cylinder to display the kinetic energy in the system. Even though this is quite a simple dynamical system, it can be used to consider and illustrate several simple but nevertheless fundamental aspects of control.

## V. CONTROL

The goal of control is to bring a certain quantity close to a desired goal. In the context of the model presented above, and given a controller object `c`, the introduction of such a controller can be modeled as follows:

```

// Goal is spring length at rest
c.g [=] s1.l+s2.l;

// Value is actual spring length
c.v [=] m1.p-m3.p;

// Add c.f
m1.f [=] s1.f1 + c.f;
m2.f [=] s1.f2 + s2.f1;

// Subtract c.f
m3.f [=] s2.f2 - c.f;

```

In this model the goal value for the controller is to have the length of the system be the same as the natural lengths of the two springs. The quantity that we wish to control is the position of the first mass minus the position of the third one. The way we will achieve that is to take a force value `f` that is generated by the controller and apply it to both sides of the system that we have constructed, but in opposing directions.

Now the question that remains is how the controller `c` should compute its output force `f` given the goal `g` and measured value `v`. This is a prototypical question in the design of control systems, and that can be approached in a variety of different ways. Three of the most basic types of controllers are 1) proportional feedback, 2) proportional/differential feedback, and 3) proportion/integral/differential feedback. The first type can work for systems without inertia, or that have intrinsic ways of dissipating inertial energy. It can be modeled as follows:

```

class force_controller_p (k_p)
  private g=[0,0,0]; v=[0,0,0];
           f=[0,0,0]
  end
  f [=] k_p * (g-v)
end

```

The force `f` computed is directly proportional (hence the name) to the difference between the goal `g` and current value `v` of the quantity that we want to control. The higher the

constant `k_p`, the higher the force that will be applied for the same amount of difference (or “error”) between the goal value and the current value.

If the system has inertia or does not dissipate the extra energy introduced by the control force, it might oscillate indefinitely as a result of the proportional control. To deal with this problem, a slightly more sophisticated controller that can also add a force opposing the direction of the motion (or rate of change) of the value being measured. Such a proportional/differential (PD) controller can be defined as follows:

```

class force_controller_pd (k_p,k_d)
  private
    g=[0,0,0]; v=[0,0,0]; s=[0,0,0];
    f=[0,0,0]
  end
  f [=] k_p * (g-v) - k_d*s
end

```

Note that this controller has an extra field `s` that should be provided from outside the object to serve as the speed reading that should affect the final force `f`.

An interesting feature of these two controllers is that they do not keep track of history. We may wish to build a controller that exerts a higher force only after a weaker force has been tested for some time. This can be helpful, for example, if there are external constant forces (such as gravity) acting on our system, and we do not know their precise quantity ahead of time. This type of behavior can be achieved by a proportional/integral/differential (PID) controller such as the following:

```

class force_controller_pid (k_p,k_i,k_d)
  private
    g=[0,0,0]; v=[0,0,0]; s=[0,0,0];
    f=[0,0,0]; i=[0,0,0]; i'=[0,0,0]
  end
  f  [=] k_p*(g-v) + k_i*i - k_d*s;
  i' [=] (g-v)
end

```

The variable `i` is being used to integrate the difference between the goal `g` and the value `v` over time, so, no extra inputs are needed.

Using the formalism presented so far, it is easy to simulate and visualize the several instances of the 3-mass/2-spring example showing both the behavior of the mass and the energy of the system with different controllers. The experiment shows that a P controller will not dissipate any energy and therefore will not stabilize the system, and that in fact at times it will add energy to the system and at others absorb energy from it. In fact, this example motivates formally analyzing this system to show that this controller will function essential as simply another spring between the two extreme masses. The PD controller will suffice in stabilizing the system quickly, and this will be clear from the height of the bar representing the energy in the system.

## VI. DISTURBANCES

To enhance the value of an experiment, whether it is a physical or a virtual experiment, it can be useful to introduce various sources of disturbance into the system. At least for preliminary experimentation, it can be sufficient to model such disturbances as autonomous sources of various forces. A simple example that can be used with the examples presented in this paper is as follows:

```
class force_disturbance (k)
  private t=0; t'=0; t''=0; f=[0,0,0] end
  t' [=] 4; f [=] k*[sin(t), cos(t),
                    sin(2*t+cos(3*t)))]
end
```

This example generates a circular motion in two dimensions, and a mildly erratic oscillation in the third dimension. The  $k$  parameter is used to determine the amplitude of the behavior. One can imagine further parameterizing the object with a frequency or with time-varying mixing of the signals along the different axes. To determine what type of disturbance is most useful for a particular class of problems requires experimental analysis and validation of the models. For the purposes of this example, the above model suffices to give us some confidence that our controllers did not just work for the particular parameters that we used to test the various systems on.

## VII. RIGID BODY DYNAMICS

With basic particle dynamics and control concepts under our belt, we are ready to start looking at the rigid body dynamics. A key feature of this level of analysis of mechanical systems is that we start to take into account both translational and rotational effects. It is also a useful level to illustrate some of the benefits of using vector algebra to model and reason about the dynamics of systems. For example, vector algebra can often allow us to think about problems in 2D and then have the results generalize naturally to 3D. For example, consider a rod which holds apart two masses (of  $m/2$  each) at a given distance (visualized in Figure 3). Now imagine that there are force vectors  $p$  and  $q$  acting on each end. What is the resulting acceleration on the system? The following class models the dynamics of such a rod:

```
class rod (m,p0,q0,D)
  private
    length = norm(p0-q0);
    p = p0; sp=[0,0,0]; q = q0; sq=[0,0,0];
    axis = (p0-q0)/norm(p0-q0);
    axis'=[0,0,0]; axis''=[0,0,0];
    core = (p0+q0)/2;
    core' = [0,0,0]; core'' = [0,0,0];
    fp = [0,0,0]; fq = [0,0,0];
    fp_axis = [0,0,0]; fp_orth=[0,0,0];
    fq_axis = [0,0,0]; fq_orth=[0,0,0];
    c = create_dumbbell (1,1,D);
  end
  fp_axis [=] dot(fp,axis)*axis
    / norm(axis);
```

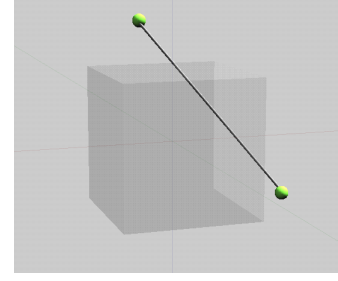


Fig. 3. The 3D visualization generated by the rod class.

```
fp_orth [=] fp - fp_axis;
fq_axis [=] dot(fq,axis)*axis
    / norm(axis);
fq_orth [=] fq - fq_axis;
core'' [=] (fp + fq)/m;
axis'' [=] 2*(fp_orth-fq_orth)
    / (m*length);
p [=] core
    + (axis * (length/2)/norm(axis));
q [=] core
    - (axis * (length/2)/norm(axis));
sp [=] core' + axis' * (length/2);
sq [=] core' - axis' * (length/2);
c.p [=] p; c.q [=] q;
end
```

It is easy to combine this system with a controller that works to move the point  $p$  to a predetermined location. An interesting question is what happens to the point  $q$  during the process, and whether that can also be controlled as well. This problem provides a natural starting point to study challenging questions such as the control of an inverted pendulum in 3D.

## VIII. DISCRETIZATION AND QUANTIZATION

The one aspect of controllers that we have not captured in the models presented above is that controllers are generally implemented by digital computers. The most obvious new issues that result from this implementation strategy are discretization (in time) and quantization (in the representation of physical quantities). Both effects can be concisely expressed in Acumen. To model discretization, the key mechanism that is needed is to define a local clock and to only allow actions to be performed (or to be observed) at clock transitions. The following class models a PID controller (like the one presented above) with discretization and quantization effects.

```
class force_controller_pid_d
  (k_p,k_i,k_d,period)
  private g=[0,0,0]; v=[0,0,0]; s=[0,0,0];
    f=[0,0,0]; t=0; t'=0;
    i=[0,0,0]; i'=[0,0,0]
  end
  t' [=] 1;
  if (t>period)
    t=0;
    f [=] k_p*(g-v) + k_i*i - k_d*s;
  end;
```



```

i' [=] (g-v)
end

```

The variables  $t$  and its derivative  $t'$  are used in a manner similar to what was done at the start of this paper to generate an interesting signal for `moving_sphere`. Here we do two new things with the variable  $t$ . The first is that we have a conditional statement based on this variable that waits until ( $t > \text{period}$ ). The parameter `period` models the time it takes the particular microprocessor that implements our controller to produce the new value of the result of the controller. Once the condition is true the first thing we do is to reset the counter. The second is that we reset its value to 0 using the statement “ $t=0$ ” as soon as that condition is true. In addition to this reset, the conditional also allows the equation for variable  $f$  in the original model to take effect only for that instant when  $t$  has surpassed the value of `period`. Because no other definition is given for this value until this event occurs again (at the start of the next period), the value  $f$  remains constant until that change occurs.

With this model, it is easy to illustrate that as the sampling period goes up, the system that we are trying to control can become unstable.

Discretization can be modeled by adding another integer (or fractional) value that is updated (by either a whole integer or a fraction, depending on the quanta) when an underlying continuous value goes outside the range represented by the current quanta.

## IX. DISCUSSION

Developing the examples used in this paper points out several possible improvements on the current formalism may be necessary. In this section, we briefly point out ones that appear to be particularly compelling.

First, when we compose several examples together, the simulation can slow down. The formalism is currently implemented as a purely functional interpreter in Scala. This serves well the goal of having a well-defined semantics. However, we expect that there will be significant opportunities for improvement in terms of performance.

Second, we have also noticed that numerical stability can be a concern, even for the simple examples presented here. The current implementation uses a forward Euler integrator to simulate the continuous behavior. While this is generally considered to be a simplistic numerical method, it is surprising that it is problematic even for the simple examples used in this paper. To address this problem without becoming dependent on a particular numerical method, we are investigating the use of interval and enclosure-based methods to define a semantics for the formalism.

Third, the examples also illustrate that the language can benefit from improved syntactic support for several different features such as: A) variable declaration and initialization often seems redundant and/or verbose, B) quantization for vector-valued variable is currently verbose, C) discretization could benefit from introducing syntactic sugar for clocks, D) embedded software could be easier to model if it can be written directly in a form similar to traditional code.

## X. CONCLUSIONS AND FUTURE WORK

In this paper we used a small domain-specific modeling formalism aimed at hybrid systems to express a range of basic aspects of robot cyber-physics. By doing so we are able to illustrate how such a formalism can be a useful basis for learning and communicating about such concepts. At the same time, we hope that these examples help communicate the richness and the power of this formalism despite its small size.

In future work, we would like to conduct similar studies to determine whether the same small formalism used here would suffice for expressing other aspects of robotic cyber-physics, including: more sophisticated control laws, models for joint and link composition, collision detection, impact dynamics, a validated model of a multi-link robot, and a validated model of a team of cooperating robots. We anticipate that the improvements suggested in the discussion section will be important for expressing these aspects naturally and concisely.

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## REFERENCES

- [1] L. Carloni, R. Passerone, A. Pinto, and A. Sangiovanni-Vincentelli, “Languages and tools for hybrid systems design,” *Foundations and Trends in Design Automation*, vol. 1, no. 1, pp. 1–204, 2006.
- [2] J. Bruneau, C. Consel, M. O’Malley, W. Taha, and W. M. Hannourah, “Virtual testing for smart buildings,” in *Proceedings of the 8th International Conference on Intelligent Environments (IE’12)*, Guanajuato’s, Mexico, 2012.
- [3] J. Jensen, D. Change, and E. Lee, “A model-based design methodology for cyber-physical systems,” Istanbul, Turkey, Jul. 2011.
- [4] E. Allen, D. Chase, J. Hallett, V. Luchangco, J.-W. Maessen, S. Ryu, G. L. Steele Jr., and S. Tobin-Hochstadt, “The fortress language specification,” Technical report, Sun Microsystems, Inc., 2007.
- [5] Y. Zhu, E. Westbrook, J. Inoue, A. Chapoutot, C. Salama, M. Peralta, T. Martin, W. Taha, M. O’Malley, R. Cartwright, A. Ames, and R. Bhattacharya, “Mathematical equations as executable models of mechanical systems,” in *Proceedings of the First ACM/IEEE International Conference on Cyber-Physical Systems*, Stockholm, Sweden, 2012.
- [6] W. Taha, P. Brauner, Y. Zeng, R. Cartwright, V. Gaspes, A. Ames, and A. Chapoutot, “A core language for executable models of cyber physical systems (preliminary report),” in *Proceedings of The Second International Workshop on Cyber-Physical Networking Systems (CPNS’12)*, Macau, China, Jun. 2012.
- [7] “Acumen web-site,” [www.acumen-language.org](http://www.acumen-language.org).
- [8] P. Fritzson and P. Bunus, “Modelica-a general object-oriented language for continuous and discrete-event system modeling and simulation,” in *SS ’02: Proceedings of the 35th Annual Simulation Symposium*. Washington, D.C., USA: IEEE Computer Society, 2002, p. 365.
- [9] W. Taha, “Lecture notes on cyber-physical modeling,” Available online from [www.effective-modeling.org/p/teaching.html](http://www.effective-modeling.org/p/teaching.html), Sep. 2012.
- [10] Y. Zeng, “Lightweight three-dimensional visualization for hybrid systems simulation,” Master’s thesis, Halmstad University, Halmstad, 2012.